https://doi.org/10.1142/9789811221903_0001

Chapter 1

New Trends and Recent Advances — An Introduction

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Enormous progress has been made in the area of nonlinear dynamics, particularly in chaos and complex phenomena during the recent years. This introductory chapter presents an overview of the contributions to this book across a wide range of topics covering chaotic, nonlinear and complex dynamics. This unique book presents advances made in theory, analysis, numerical simulation, experimental realization and promising novel practical applications on various topics of current level interest on chaos and related fields in nonlinear dynamics.

1.1. Introduction

The research monograph *Recent Trends in Chaotic, Nonlinear and Complex Dynamics* is in honor of Professor Miguel A. F. Sanjuán on the occasion of his 60th anniversary. It contains a collection of original research papers presented in the special session devoted to the celebration of the 60th birthday of Miguel A. F. Sanjuán in the *15th International Conference on Dynamical Systems — Theory and Applications* (*DSTA 2019*).

The present book constitutes an invaluable reference source of recent trends on chaotic and complex dynamics for researchers and newcomers to the field of nonlinear dynamics. Particularly, it covers such areas as the complex behavior of the dipole-segment with equal masses and arbitrary rotation, the dynamics of the retina system, stability of anti-bunched buses and local unidirectional Kuramoto oscillators, the role of the noise in the chaotic intermittency, control of quasi-passive dynamic walker based on entrainment effect, numerical errors and exact orbits of quadratic maps, feedback delay control of the complex Ginzburg–Landau equation with local and nonlocal delayed perturbation, the features of the physics-inspired particle swarm optimization algorithm, stochastic sensitivity analysis of noise-induced phenomena in discrete systems, electromechanical models of micro- and nanoresonators, enhanced vibrational resonance induced by an amplitude modulated force, generation of self-excited and hidden attractors in multistable systems, modal asynchronicity in pre-stressed continuous beams, collective sustained oscillations in complex systems, weak decay of autocorrelations in periodic barrier billiards, the interaction of vortices in stratified incompressible flows and fractal basin of attraction in planar switched systems.

Apart from the present chapter the book contains 18 chapters. These chapters broadly cover the following three aspects:

Part I. Fundamental Results in Nonlinear Dynamics

Chapter 2. Enhanced vibrational resonance by an amplitude-modulated force by Chinnathambi *et al.* [1].

Chapter 3. Generation of self-excited and hidden multiscroll attractors in multistable system by Escalante-González and Campos [2].

Chapter 4. Dynamics of the dipole-segment with equal masses and arbitrary rotation by Elipe *et al.* [3].

Chapter 5. The interaction of two vortices near a boundary in rotating stratified incompressible flows by Carton *et al.* [4].

Chapter 6. Modal asynchronicity in pre-stressed continuous simply-supported beams with transversal and rotational oscillators by Ribeiro *et al.* [5].
Chapter 7. On the weak decay of autocorrelations in periodic barrier billiards by Courbage [6].

Part II. Stochastic Dynamics, Fractal Structure Analysis and Numerical Errors

Chapter 8. Stochastic sensitivity analysis of noise-induced phenomena in discrete systems by Bashkirtseva and Ryashko [7].
Chapter 9. The role of noise in chaotic intermittency by del Rio and Elaskar [8].
Chapter 10. Fractal structures in a binary Schwarzschild black hole system by de Souza Filho *et al.* [9].
Chapter 11. Characterizing fractal basin of attraction in planar switched systems by Zhang [10].
Chapter 12. Numerical errors and exact orbits of quadratic maps by Gallas [11].

Part III. Recent Trends in Applications of Chaotic and Complex Systems

Chapter 13. Electromechanical models of micro- and nanoresonators by Lukin *et al.* [12].
Chapter 14. The retina as a dynamical system by Cessac [13].
Chapter 15. Collective sustained oscillations in complex systems by Zheng [14].
Chapter 16. Stability of anti-bunched buses and local unidirectional Kuramoto oscillators by Yue Chew *et al.* [15].
Chapter 17. Feedback delay as a control tool: the complex Ginzburg–Landau equation with local and nonlocal delayed perturbations by Casal and Díaz [16].
Chapter 18. Nonlinear analysis and control for quasi-passive dynamic walker based on entrainment effect by Li *et al.* [17].
Chapter 19. Physics-inspired swarm optimization: the general algorithmic search by Hernández *et al.* [18].

1.2. Fundamental Results in Nonlinear Dynamics

Resonance refers to a realization of a maximum response of a system induced by means of one or more external forces. Nonlinear systems are capable of exhibiting a variety of resonances depending on the nature of the external forces. Vibrational resonance (VR) induced by a biharmonic force is a fundamental phenomenon with a variety of practical applications in physics, engineering, and biology. Part I begins with the chapter on the VR by an amplitude modulated force by Chinnathambi, Rajasekar and Sanjuan [1]. First, the occurrence of VR is analyzed in the paradigmatic Duffing oscillator with a single-well potential and the external force consisting of one low-frequency and one high-frequency component. Analytical expressions and theoretical predictions for the response amplitude are verified through numerical simulation. Next, the authors show that a higher number of resonance peaks, hysteresis and a jump phenomenon are due to the presence of the second high-frequency component in the external forcing. Finally, the system with a double-well potential is investigated. The mechanism of the enhanced VR is explained by using the phase portrait and the width of the orbit.

Escalante-González and Campos present recent approaches to generate self-exited and hidden multiscroll attractors in multistable systems in Chapter 3 [2]. A vector field is defined by affine linear systems such that each equilibrium is a saddle point. The space is partitioned in hyperbolic set and the generated self-excited multiscroll attractors are based on heteroclinic orbits. The authors report an interesting phenomenon when the equilibria are separated by pairs and the coexistence of different self-excited double-scroll attractors arise. A class of systems with 1D, 2D and 3D scrolls attractors are generated around multiple self-excited attractors. This class of systems presents the coexistence of self-excited double-scroll chaotic attractors and a hidden multiscroll attractor.

Many bodies in the solar system have irregular shape. Planned missions to orbit around and land on such bodies require to model the gravity fields of such bodies. In Chapter 4, Elipe, Abad and Ferreira investigate the dynamics of the dipole-segment system (DSS) (two spherical masses placed at the endpoints of a finite massive straight element) [3]. The gravity function of the DSS is expressed in closed form, without a series expansion. The authors compute the equilibria, their stability, and find families of periodic orbits. It appears that the dynamics of the DSS is very rich, and it opens the way to new studies from the dynamical systems point of view.

Chapter 5 by Carton, Tchuenkam and Vic analyze the interaction of two vortices near a boundary in rotating stratified incompressible flows [4]. Point vortex dynamics is applied to the study of oceanic vortices. The evolution of the North Brazil current rings, idealized as point vortices, in a stratified rotating fluid, along a straight or angular wall, representing the northern coast of South America is analyzed. It is demonstrated that the point vortex model with initial two-vortex configuration, though highly idealized, can exhibit time evolution qualitatively comparable with reality. Point vortex models and mathematical models of vortex dynamics are thus an efficient tool for the exploration of the behavior of oceanic vortices.

Analysis of modal asynchronicity in pre-stressed continuous simply-supported beams with transversal and rotational oscillators is presented in Chapter 6 by Ribeiro, Lenci and Mazzilli [5]. They show that even very simple models, with few degrees of freedom can exhibit the effect of modal localization. However, the situation becomes much more complicated in continuous systems. This chapter demonstrates that modal localization occurs in a pre-stressed continuous simply-supported beam after transversal and rotational oscillators are added, and then parameter fine-tuning is performed. These results pave the way towards the construction of novel vibration controllers and efficient energy harvesters.

Chapter 7 by Courbage is concerned with the periodic barrier billiards problem [6]. The author shows that the process of reversing the direction of velocity in barrier billiards can have a singularly continuous spectrum. These long memory effects can coexist with other strange aspects, as an exponential decay of correlations in zero entropy maps. Further, the analysis is focused on the investigation of such dynamical systems in the gap between chaos and order.

1.3. Stochastic Dynamics, Fractal Structure Analysis and Numerical Errors

Investigation of features of noise-induced phenomenon are of great applications in various contexts. The first chapter of Part II, Chapter 8 by Bashkirtseva and Ryashko reports noise-induced phenomena in map-based stochastic systems [7]. It is demonstrated that the interrelation of nonlinearity and stochasticity often leads to new phenomena that are not observable in the original deterministic models. The authors discuss the novel approach for the study of noise-induced phenomena based on the stochastic sensitivity method and the confidence domain method. The stochastic sensitivity analysis of equilibria, cycles, closed invariant curves, and chaotic attractors is presented in this chapter. The suggested analytical approach is illustrated for diverse stochastic phenomena in discrete systems with quadratic map and coupling.

Further, on the effect of noise, in Chapter 9 the role of the noise on the intermittency is presented by del Rio and Elsakar [8]. Intermittency can be characterized by spontaneous transitions between laminar and chaotic dynamics. This chapter presents a thorough description of the effects of the additive noise to systems that have intermittency. The theory of intermittency is extended in order to reproduce statistic behavior far from classical predictions. The authors show that the effects of the noise on the chaotic region also affects the statistical dynamics in the laminar region. A good agreement is demonstrated between the proposed theory and the numerical simulations. Finally, the new intermittency effects found in continuous systems are discussed.

The dynamics of open Hamiltonian systems with chaotic scattering can be characterized by fractal structures. For example, a light ray approaching a pair of super massive black holes can diverge to infinity, fall down into one of the black holes or can orbit around the black hole pair as periodic orbits. In Chapter 10, Filho, Mathias and Viana investigate the escape basins and their fractal boundaries [9]. The fractality of the escape basin is investigated by using a scattering map. The fractal nature of the structure of escape basin is quantitatively investigated by computing the basin entropy and the basin boundary entropy. The authors show that the distance between the black holes is an important parameter impacting the complexity of the basin structure.

Furthermore, on fractal structures, Zhang devoted Chapter 11 for the fractal and Wada basin dynamics of the switched systems [10]. These systems comprise an important class of dynamical systems that consist of a finite number of subsystems and a switching rule between these subsystems. This chapter investigates the existence of fractal and Wada basin boundaries to the generally discrete-time switched systems with time-dependent switching. Analytical tools for the characterization of the properties of fractal basin boundaries for planar switched systems (with time-dependent switching) are constructed. It is demonstrated that the basin boundaries of an auxiliary dynamical system (related to the switched system) provide an evidence of the Wada basin boundaries for switched systems. Wada basin boundaries are used to provide the insight into the qualitative unpredictability and uncertainty of the switched systems.

The accumulation of errors and the numerical noise poses serious problems when performing large number of iterates in computational simulations of nonlinear systems. Gallas considers two problems related to computational analysis of dynamical systems in Chapter 12 [11]. The first problem is focused on the impact of small numerical errors in calculations of orbital points of the two-dimensional Hénon map. Numerical errors are determined by measuring the distance between the points of departure and return in the original and the inverse maps. Secondly, exact analytical results are given for the one-dimensional quadratic map in the fully chaotic partition generating limit. The presented algorithm allows to extract precise coordinates of points belonging to periodic orbits. Such techniques provide valuable tools which help to gauge the propagation of errors in computational analysis of chaotic nonlinear maps.

1.4. Recent Trends in Applications of Chaotic and Complex Systems

Part III begins with Chapter 13 by Lukin, Privalova, Popov, Skubov and Shytukin on the study of the periodical oscillations of mono- and multilayer nanocapacitors located in the electric field [12]. The authors observe that the time of overcharge of nanocapacitor is much less than the period of excited oscillations. One of possible applications of graphene nanoresonators is using of these devices as detectors of the mass of nanoparticles adherent on the graphene layer. The adherence of the particle (biological cell or even molecules) to the elastic surface changes its resonance frequency. This effect provides means for the determination of the mass of the nanoparticle.

Chapter 14 deals with the retina as a high-dimensional system [13]. The specific and hierarchical structure of the eye retina allows it to pre-process visual information, at different scales, in order to reduce redundancy and increase the speed, efficiency and reliability of visual response. In this chapter, Cessac demonstrated that the theory of dynamical systems, the theory of bifurcations, and the ergodic theory can provide useful insights on retinal behavior and dynamics. The retina is investigated as a high-dimensional, non-autonomous dynamical system, layered and structured, with non-stationary and spatially inhomogeneous entries. Finally, the dynamics of the system is generalized to more general perspectives.

Collective dynamics of spatiotemporally coupled elements can be considered as the emergence of the order from microscopic self-organization at the macroscopic scale. Zheng in Chapter 15 explores the mechanism of collective self-sustained oscillations in networks of coupled non-oscillatory units in [14]. It is demonstrated that the network topology plays a significant role in the dynamics of excitable networks and gene regulated networks. This chapter shows that the topology reduction is the key procedure in accomplishing the dimension-reduction description of a complex system.

Urban planning plays a crucial role in empowering development and sustainability of major cities around the world. In Chapter 16, Chew, Saw and Pang introduce a no-boarding policy as a form of kicking force to achieve a stable staggered state in the bus loop system [15]. The bus bunching phenomenon is related to the effect of synchronization of Kuramoto phase oscillators with local unidirectional coupling. It is shown that stable anti-bunched states can exist (without any additional kicking force) if the number of oscillators is at least five. This similarities between the bus loop system and the local unidirectional Kuramoto oscillators provide the insight on how the bus loop system can remain staggered.

In Chapter 17, the role of feedback delay as a control tool in the complex Ginzburg–Landau equation with local and nonlocal delayed perturbations is discussed by Casal and Díaz [16]. The authors consider the global delayed problem in which two real parameters, namely, the feedback intensity and the delay time play a fundamental role. The basic questions of the existence and uniqueness of solutions to the associate initial-boundary value problem with initial past history are rigorously discussed. Novel results on the stabilization of the uniform oscillations for the complex Ginzburg–Landau equation by means of global delayed feedback are presented. Further, several bifurcation effects produced by the delay time in the behavior of the complex Ginzburg–Landau equation are analyzed.

In classical passive walking, the locomotion power comes solely from the gravity. The walking cycles are sustained on a gentle downhill and the system converges to a limit cycle, in which gravitational potential energy and energy loss due to foot-landing cancel each other. In Chapter 18, Li, Tokuda, Asano and Yan present an overview on the passive dynamical walking as a typical example of nonlinear dynamics in the field of robotics [17]. A control mechanism which positively utilizes the intrinsic walking property, while preserving its efficiency is introduced. The authors demonstrate both numerically and experimentally that the well–known Arnold tongues are observed as a typical entrainment property of the limit cycle walker in two-dimensional parameter space spanned by the wobbling forcing and frequency.

In Chapter 19, Hernández, Duran and Amigó present a physics-inspired particle swarm algorithm for global optimization [18]. A stochastic, single-objective method that evolves a swarm of agents in search of global extremum is designed and discussed. The governing principles of the presented optimization algorithm are based on physical processes observable in dynamical systems. Numerical simulations are performed with 31 test functions with dimensions ranging between 2 and 45. It is shown that the general algorithmic search outperforms such global optimization algorithms as basin hopping, Cuckoo search, and differential evolution algorithms.

1.5. Conclusions

This research monograph covers new and novel emerging topics and expounds the state-of-the-art advancements in the field, including theory, analysis, numerical simulations, novel applications and future directions of research on chaos and complex system dynamics. It presents a comprehensive picture of recent developments in chaos and complex dynamics that is useful to a wide audience including newcomers, lecturers and researchers in physics, engineering, and applied mathematics.

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