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Numerical procedure for fluid flow in a pipe performing transverse oscillations

M. Ragulskis[1,*,†], A. Bubulis[2] and K. Ragulskis[3]

[1]*Department of Mathematical Research in Systems, Kaunas University of Technology, Kaunas, Lithuania*
[2]*Institute of Piezomechanics, Kaunas University of Technology, Kaunas, Lithuania*
[3]*Lithuanian Academy of Sciences, Lithuania*

SUMMARY

Numerical procedure for the analysis of fluid flow in a tube performing transverse oscillations is developed. The formulation of the problem is presented in differential equation form and a finite element model is developed leading to the first-order matrix differential equation. The fluid flow model incorporates transverse oscillation of the boundary through the convective inertia terms. Modal decomposition of the solution is performed and a technique for numerical solution of the finite element problem incorporating parametric vibrations is developed. Numerical results provide insight into the problem of fluid flow control by transverse vibrations of the tube. Copyright © 2006 John Wiley & Sons, Ltd.

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1. INTRODUCTION

In many industrial applications mechanical structures like heat exchanger tubes, gas pipelines, fluid conveying and dosing pipes are subjected to complex flows causing vibrations and possible damage [1–3]. Various damping and control strategies are used to quench these vibrations or to control the conveyance of the fluid [4, 5]. In such situations transverse tube vibrations play a dominant role [6, 7].

The objective of this paper is to analyse the influence of the forced transverse vibrations of a straight tube to the fully developed flow in that tube. Numerous computational techniques could be applicable for analysis of the described system. Usually, those techniques involve three-dimensional finite element formulations of the fluid and computational analysis of transient flow processes [8, 9].

*Correspondence to: M. Ragulskis, Department of Mathematical Research in Systems, Kaunas University of Technology, Studentu 50-222, Kaunas, Lithuania.
†E-mail: minvydas.ragulskis@ktu.lt

The main objective of this paper is to propose an effective numerical technique, which is much faster and simpler than the classical techniques.

Numerical model for the analysis of fluid flow in a pipe performing longitudinal vibrations is presented in Reference [10]. The problem of fluid flow control exploiting transverse vibrations of the boundary requires essential modifications of the algorithm described in Reference [10] where harmonic balance method [11] was used to solve the physically non-linear problem (non-Newtonian fluid flow). In this paper, the problem is geometrically non-linear (Newtonian fluid, transverse boundary oscillation). The problem is formulated as a linear parametric problem and harmonic balance method is developed for efficient analysis of steady state parametric vibrations.

First, a mathematical model of the system is developed in the form of differential equation with partial derivatives incorporating the boundary conditions. Flow excitation by transverse vibrations of the tube is represented through the convective acceleration terms. The excitation velocities are assumed to be equal in the whole cross-section area and are the functions of time only. Finite element model of the system is obtained, what leads to the first-order matrix differential equation. Its eigenpairs are calculated and the modal decomposition of the solution is performed. Finally, the steady state solution is sought using the proposed procedure based on the theory of parametric vibrations [11].

The obtained results provide insight into the process of vibration-based fluid flow control.

2. NUMERICAL ALGORITHM

Cartesian co-ordinate system is defined where the z-axis is parallel to the axis of the tube. It is assumed that the cross section of the tube does not vary with the z co-ordinate and the velocity of fluid flow in the direction of the flow is the function of the co-ordinates of the cross section and time, while the components of velocity in the plane of the cross section are given functions of time only:

$$u = u(t), \quad v = v(t), \quad w = w(x, y, t) \tag{1}$$

where u, v, w denote the velocity components in the directions of the Cartesian orthogonal axes of co-ordinates; t is time (Figure 1). Such assumptions are applicable for the analysis of fully developed flow (assuming that the tube is long and the analysed cross section is far from the ends of the tube).

The dynamic equilibrium equation in the direction of the z-axis takes the form (taking into account the full derivative of w):

$$\frac{\partial}{\partial x}\left(\mu\frac{\partial w}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial w}{\partial y}\right) - \frac{\partial p}{\partial z} + \rho g = \rho\frac{\partial w}{\partial t} + \rho u\frac{\partial w}{\partial x} + \rho v\frac{\partial w}{\partial y} \tag{2}$$

where p denotes the pressure; μ is the viscosity of the fluid; ρ is the density of the fluid; g is the acceleration of gravity; $\partial p/\partial z$ is the gradient of the pressure in the direction of the z-axis and it is assumed constant along the tube.

The boundary condition of the wall takes the form

$$-\mu\frac{\partial w}{\partial n} = \alpha w \tag{3}$$

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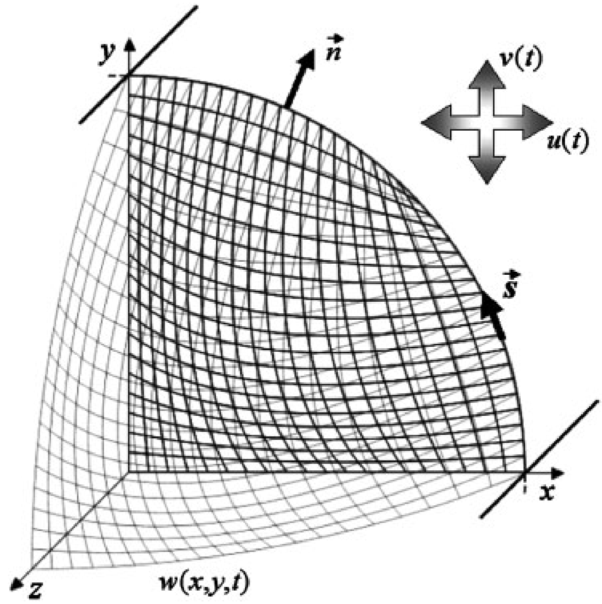

Figure 1. The principal model of the system.

where n is the outward normal vector to the boundary of the cross section of the tube and α is the coefficient of slippage (sliding friction between the fluid and the surface of the tube). Such a boundary condition of Robin type is common in the analysis of transport of various oils and suspensions [12]. The coefficient α can be chosen on the basis of correlation of the calculated and experimentally measured velocity profiles at the boundary of the tube. It can be noted that conventional non-slippage boundary condition can be incorporated by selecting a large value of α.

It is assumed that the boundary performs harmonic one-directional oscillations in the plane of the cross section in an arbitrary prescribed transverse direction. The components of the vibration velocity vector are expressed as

$$\begin{aligned} u &= a \sin \omega t \\ v &= b \sin \omega t \end{aligned} \tag{4}$$

where a and b are the amplitudes of harmonic oscillations of the boundary's velocity in the direction of x- and y-axes; ω is the angular frequency. Thus, the tube performs non-elliptic, one-directional oscillations. The displacements of the tube can be calculated integrating Equation (4): $-(a/\omega)\cos \omega t$ and $-(b/\omega)\cos \omega t$. Therefore, the amplitude of the kinematic excitation is $\sqrt{a^2+b^2}/\omega$ and the angle φ between the direction of oscillations and the x-axis is $\varphi = \tan^{-1}(b/a)$.

The cross section of the flow is meshed using the finite element approximation. The resulting matrix differential equation

$$[C]\{\dot{\delta}\} + [K]\{\delta\} = \{F\} \tag{5}$$

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is obtained on the basis of the Galerkin method of weighted residuals [12, 13]. Matrixes of the finite element method (FEM) take the following form:

$$
\begin{aligned}
[C] &= \int\int [N]^{\mathrm{T}} \rho [N] \,\mathrm{d}x\,\mathrm{d}y \\
[K] &= \int\int [B]^{\mathrm{T}} \mu [B] \,\mathrm{d}x\,\mathrm{d}y + \oint [M]^{\mathrm{T}} \alpha [M] \,\mathrm{d}s \\
\{F\} &= \int\int [N]^{\mathrm{T}} \left(\rho g - \frac{\partial p}{\partial z}\right) \mathrm{d}x\,\mathrm{d}y - \int\int [N]^{\mathrm{T}} \rho [u;\, v][B]\{\delta\} \,\mathrm{d}x\,\mathrm{d}y
\end{aligned}
\tag{6}
$$

where $\{\delta\}$ is the vector of nodal velocities. The upper dot in Equation (5) denotes differentiation with respect to time; s is the boundary line of the cross section of the flow; $[N]$ is the row vector of the shape functions of the finite element in the cross section of the flow; $[B]$ is the matrix of the derivatives of the shape functions (the first row with respect to x and the second row with respect to y); $[M]$ is the row vector of the shape functions of the finite element on the boundary of the cross section of the flow. Two-dimensional isoparametric 9-node quadrilateral Lagrange elements [12, 14] are used for the discretization of the cross section and one-dimensional 3-node Lagrange elements [12, 14] are used on the boundary of the cross section (the second integral in the expression of the matrix $[K]$).

First, the eigenpairs of the homogeneous system $[C]\{\dot{\delta}\} + [K]\{\delta\} = 0$ are determined. The solution of the system is expressed in the form $\{\delta(t)\} = \{\bar{\delta}_i\}\mathrm{e}^{-\lambda_i t}$, where $\{\bar{\delta}_i\}$ is the ith eigenvector of the decay of the fluid motion, and $\lambda_i > 0$ is the ith eigenvalue; $i = 1, \ldots, n$, here n is the number of degrees of freedom. Numerical calculation of the eigenpairs (eigenvectors and eigenvalues) is performed using the method of subspace iterations [14]. The vector of nodal fluid velocities is expressed as:

$$
\{\delta\} = [\Delta]\{Z\} = [\Delta] \begin{Bmatrix} z_1 \\ z_2 \\ \ldots \end{Bmatrix} \tag{7}
$$

where $[\Delta] = [\{\bar{\delta}_1\}\ \{\bar{\delta}_2\}\ \ldots]$; z_i denotes the modal coefficient for mode i. Then modal decomposition of Equation (5) is performed [10, 13]: $[\Delta]^{\mathrm{T}}[C][\Delta]\{\dot{Z}\} + [\Delta]^{\mathrm{T}}[K][\Delta]\{Z\} = [\Delta]^{\mathrm{T}}\{F\}$. It can be noted that the subspace iteration method produces orthonormal eigenvectors satisfying the conditions

$$
\{\bar{\delta}_i\}^{\mathrm{T}}[C]\{\bar{\delta}_j\} = \begin{cases} 1, & \text{when } i = j \\ 0, & \text{when } i \neq j \end{cases} \quad \text{and} \quad \{\bar{\delta}_i\}^{\mathrm{T}}[K]\{\bar{\delta}_j\} = \begin{cases} \lambda_i, & \text{when } i = j \\ 0, & \text{when } i \neq j \end{cases}
$$

So modal equations take the form $\mathrm{d}/\mathrm{d}t(z_i) + \lambda_i z_i = \{\bar{\delta}_i\}^{\mathrm{T}}\{F\}$.

The determination of the averaged surface of transverse velocities and the mass flow rate is performed by solving the following four successive problems similar to harmonic balance procedure applied for non-Newtonian fluid flow in a longitudinally vibrating tube [10]. Nevertheless, the proposed technique substantially differs from the one presented in Reference [10] first of all due to the differences in the formulation of the problem.

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Static solution. The static (constant in time) solution is obtained assuming that the term $-(\partial p/\partial z) + \rho g$ is constant in time and in-plane velocities are negligible ($u = v = 0, \{\delta\} = 0$). Thus, the static load is formed according to the following equation:

$$\{F_0\} = \int\int [N]^{\mathrm{T}} \left(\rho g - \frac{\partial p}{\partial z}\right) \mathrm{d}x\, \mathrm{d}y \tag{8}$$

This requires the solution of the system of linear algebraic equations $[K]\{\delta_0\} = \{F_0\}$. This solution is sought using modal decomposition. Denoting

$$\begin{Bmatrix} f_1^0 \\ f_2^0 \\ \dots \end{Bmatrix} = [\Delta]^{\mathrm{T}}\{F_0\} \quad \text{and} \quad \{\delta_0\} = [\Delta] \begin{Bmatrix} z_1^0 \\ z_2^0 \\ \dots \end{Bmatrix}$$

for each eigenmode it follows that $z_i^0 = f_i^0/\lambda_i$.

Steady state harmonic solution. Harmonic variation of the in-plane velocities is assumed while the condition $-(\partial p/\partial z) + \rho g = 0$ is being satisfied. Steady state harmonic motion of the system is obtained taking into account $\{\delta_0\}$ from the previous step and performing the solution of the modal equations

$$\frac{\mathrm{d}}{\mathrm{d}t}(z_i) + \lambda_i z_i = f_i \tag{9}$$

where f_i is the modal loading for mode i. It is assumed that $\{F\} = \{F_{\mathrm{s}}\} \sin \omega t$. Then,

$$\{F_{\mathrm{s}}\} = -\int\int [N]^{\mathrm{T}} \rho [a; b][B]\{\delta_0\}\, \mathrm{d}x\, \mathrm{d}y \quad \text{and} \quad \begin{Bmatrix} f_1^{\mathrm{s}} \\ f_2^{\mathrm{s}} \\ \dots \end{Bmatrix} = [\Delta]^{\mathrm{T}}\{F_{\mathrm{s}}\}$$

Thus, $f_i = f_i^{\mathrm{s}} \sin \omega t$, what produces $z_i = z_i^{\mathrm{s}} \sin \omega t + z_i^{\mathrm{c}} \cos \omega t$. The vectors of the sine and cosine components of the steady state motion $\{\delta_{\mathrm{s}}\}$ and $\{\delta_{\mathrm{c}}\}$ are then expressed as

$$\{\delta_{\mathrm{s}}\} = [\Delta] \begin{Bmatrix} z_1^{\mathrm{s}} \\ z_2^{\mathrm{s}} \\ \dots \end{Bmatrix}, \quad \{\delta_{\mathrm{c}}\} = [\Delta] \begin{Bmatrix} z_1^{\mathrm{c}} \\ z_2^{\mathrm{c}} \\ \dots \end{Bmatrix} \tag{10}$$

The following system of linear algebraic equations is solved for each eigenmode:

$$\begin{bmatrix} \lambda_i & -\omega \\ \omega & \lambda_i \end{bmatrix} \begin{Bmatrix} z_i^{\mathrm{s}} \\ z_i^{\mathrm{c}} \end{Bmatrix} = \begin{Bmatrix} f_i^{\mathrm{s}} \\ 0 \end{Bmatrix} \tag{11}$$

Up to this point the total velocities are $\{\delta(t)\} = \{\delta_0\} + \{\delta_{\mathrm{s}}\} \sin \omega t + \{\delta_{\mathrm{c}}\} \cos \omega t$.

Averaged transverse velocities. The load vector is obtained in the process of assembly of the following loads:

(a) the load occurring from the constant term $-(\partial p/\partial z) + \rho g$;
(b) the term occurring from the static solution and the harmonic motion including the prescribed sinusoidal variation of the velocities u and v.

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Those calculations are performed over one period of steady state harmonic motion. Equally spaced discrete time moments $\omega t = (2\pi/m)(i-1)$, $i = 1, \ldots, m$ (m is the number of discrete points) are used to approximate the time integral. Time average of load vector takes the form:

$$\begin{aligned}
\{F_1\} &= \overline{\int\int [N]^{\mathrm{T}}\left(\rho g - \frac{\partial p}{\partial z}\right) \mathrm{d}x\,\mathrm{d}y - \int\int [N]^{\mathrm{T}}\rho[u; v][B]\{\delta\}\,\mathrm{d}x\,\mathrm{d}y} \\
&= \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} \left(\int\int [N]^{\mathrm{T}}\left(\rho g - \frac{\partial p}{\partial z}\right) \mathrm{d}x\,\mathrm{d}y - \int\int [N]^{\mathrm{T}}\rho[u; v][B]\{\delta\}\,\mathrm{d}x\,\mathrm{d}y\right)\mathrm{d}t \\
&= \int\int [N]^{\mathrm{T}}\left(\rho g - \frac{\partial p}{\partial z}\right) \mathrm{d}x\,\mathrm{d}y - \frac{1}{m}\sum_{i=1}^{m}\left(\int\int [N]^{\mathrm{T}}\rho[a; b]\sin\left(\frac{2\pi}{m}(i-1)\right)[B]\right. \\
&\quad \times \left.\left\{\{\delta_0\} + \{\delta_{\mathrm{s}}\}\sin\left(\frac{2\pi}{m}(i-1)\right) + \{\delta_{\mathrm{c}}\}\cos\left(\frac{2\pi}{m}(i-1)\right)\right\}\mathrm{d}x\,\mathrm{d}y\right)
\end{aligned} \tag{12}$$

where the overbar denotes time average.

Finally, the problem is solved obtaining the averaged surface of transverse velocities using the previously described technique of modal decomposition for the solution of the system of linear algebraic equations $[K]\{\delta_1\} = \{F_1\}$.

Mass flow rate. Finally, the average mass flow rate is found by integrating over the cross-sectional area:

$$Q = \int\int \rho w(x, y)\,\mathrm{d}x\,\mathrm{d}y \tag{13}$$

where $w(x, y)$ are the averaged transverse velocities which are calculated from $\{\delta_1\}$ by using the shape functions of the appropriate finite elements. It is assumed that the averaging interval is much longer than the period of oscillations.

3. NUMERICAL RESULTS

Cross section of the tube is assumed to be a circle and one-fourth of it is analysed. The characteristics of the fluid represent a liquid-type suspension ($\mu = 0.004\,\mathrm{g/mm\,s} = 4cP$, $\alpha = 0.04\,\mathrm{g/mm^2\,s}$, $\rho = 0.001\,\mathrm{g/mm^3}$, $-\partial p/\partial z + \rho g = 0.003\,\mathrm{g/mm^2\,s^2}$). Radius of the tube is $R = 10\,\mathrm{mm}$; number of discrete time moments in a period of steady state harmonic motion used for averaging in Equation (12) is $m = 32$.

Isolines of the averaged velocities when the boundary is oscillating in the x direction are presented in Figure 2. Nodal values of the averaged cross-flow velocities are obtained as the vector $\{\delta_1\}$ by solving the system of linear algebraic equations $[K]\{\delta_1\} = \{F_1\}$ (stage of calculation of averaged transverse velocities). Then a continuous field of averaged cross-flow velocities in the cross section of the tube is interpolated from the nodal values by the shape functions of appropriate finite elements.

It is clear that without external dynamic excitation the contour lines would be concentric circles (one is to have in mind that the flow takes place due to the pressure gradient and gravity, while the

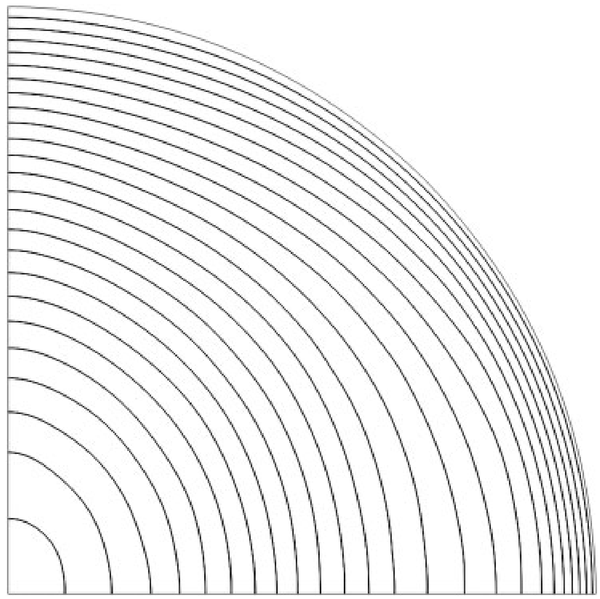

Figure 2. Isolines of averaged cross-flow velocities when the tube performs transverse oscillations in the direction of the x-axis ($a/\omega = 3\,\mathrm{mm}$; $b/\omega = 0\,\mathrm{mm}$; $\omega = 2\,\mathrm{rad/s}$).

oscillations only influence that flow). It can be seen that the map of average cross-flow velocities is altered by transverse excitation what results in lower mass flow rates. Such an effect can be explained by the presence of convective inertia terms of the fluid in the governing equation of motion.

Relationships between the mass flow rate and the amplitude of excitation at different frequencies of excitation are presented in Figure 3. One must have in mind that a is the amplitude of oscillations of the velocity (Equation (4)), while the amplitude of kinematic excitation is a/ω.

Numerical integration of the finite element equations was carried out in order to validate the proposed algorithm. Constant average velocity Newmark-type numerical integration scheme (similar to the constant average acceleration Newmark scheme [14]) was used for time integration of modal equations. One-dimensional first-order differential equation $c\dot{x} + kx = f(t)$ is solved by determining $\dot{x}(t)$ from $(c + k(\tau/2))\dot{x}(t) = f(t) - k(x(t-\tau) + \dot{x}(t-\tau)(\tau/2))$ and then calculating $x(t) = x(t-\tau) + (\dot{x}(t-\tau) + \dot{x}(t))(\tau/2)$, where τ is the time step. The results confirm the effect of decrease of average mass flow rate due to the vibrations of the tube (Figure 4).

The main advantage of the proposed technique is that the steady state regimes can be determined directly. The calculation of the instantaneous mass flow rate must be performed for each time step when performing time integration—it is used for the determination of the average mass flow rate over the last calculated period of steady state motion. Assuming that the cost of numerical integration over one time step is $\overline{R}$; the calculation of the mass flow rate is performed for each time step at a cost $\overline{r}$; m time steps in a period of excitation are used; 10 periods are required to achieve steady state motion, the cost of numerical integration can be approximated as $10m(\overline{R} + \overline{r})$.

The costs of the proposed procedure for the same problem can be approximated as follows: steady state harmonic solution—cost is $\overline{R}$ (comparable with one time step); static solution—cost

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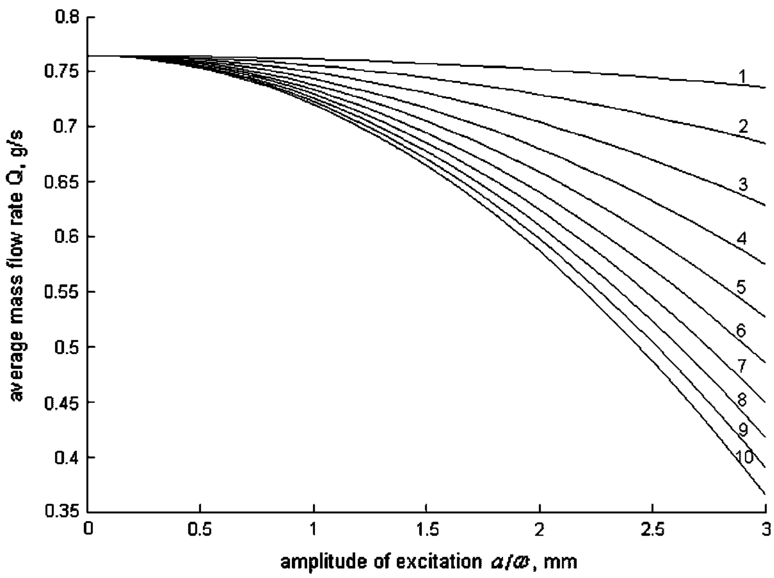

Figure 3. Relationship between the mass flow rate Q (g/s) and the amplitude of excitation a/ω (mm) at various frequencies of excitation $\omega_i = 0.2 + 0.2i$, $i = 1, 2, \ldots, 10$.

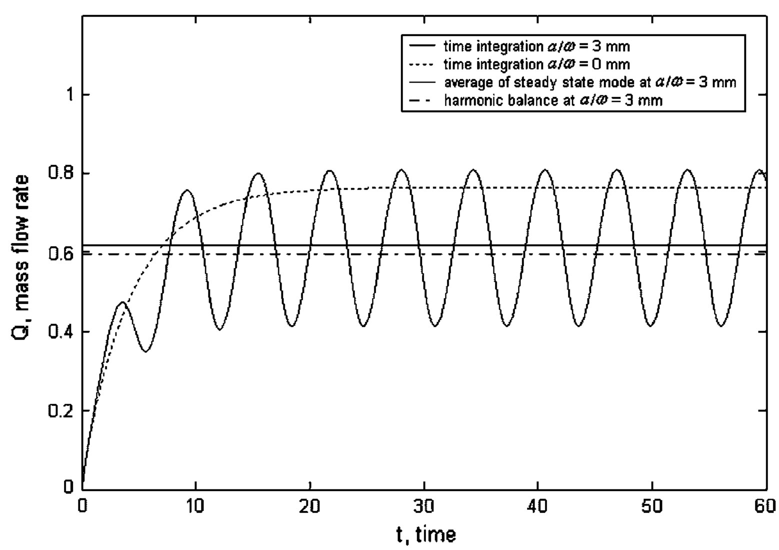

Figure 4. Mass flow rate obtained by numerical integration and by the proposed technique.

is $\overline{R}/2$ (about half of the cost of harmonic solution); determination of the averaged transverse velocities—cost is $m\overline{R}/2$; determination of the average mass flow rate—cost is $\overline{r}$. So the total cost of the proposed procedure is $(3+m)/2 \cdot \overline{R} + \overline{r}$. For reasonable meshings $\overline{r}$ can be approximated

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as $0.1\overline{R}$. Thus, the cost of the proposed procedure is about 20 times lower than that of direct time integration technique. Of course, the conclusions about the efficiency of the process depend on the method of numerical integration: higher-order numerical integration schemes enable to increase the time step, but usually require more operations per step. Thus, the presented estimates should be regarded as typical, though approximate.

4. CONCLUSIONS

The mathematical model describing the motion of fluid in a tube performing transverse vibrations is developed. The dynamical model incorporates convective inertia terms representing excitation from transversally oscillating boundary. The proposed method for the determination of the averaged velocities is much more effective compared with direct numerical integration techniques. Obtained numerical results show that transverse vibrations of the tube can control mass flow rate. Such control techniques can be incorporated into the design of spraying and dosing devices of various substances.

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