TECHNIQUES

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IDENTIFICATION OF IN-PLANE VIBRATIONS USING TIME AVERAGE STOCHASTIC MOIRÉ

Geometric moiré is a powerful experimental technique applicable for the determination of in-plane displacements of various structures. Time average geometric moiré is a technique providing insight into dynamic displacements of vibrating elastic structures. Though there exist advanced techniques for formation of moiré gratings on even microscopic surfaces, applicability of time average techniques in such cases is rather problematic first of all due to the fact that microscopic areas of macroscopic objects tend to vibrate as non-deformable bodies. Then the following question rises—would it be possible to exploit the natural, stochastic structure of the object's surface for identification of its in-plane vibrations? In case if the answer is positive, a simple cost-effective inverse engineering technique could be developed.

Classical geometric moiré[1,2] techniques have a general limitation associated with the fact that the displacements can be determined only in the direction orthogonal to the lines of the grating.[3,4] The same limitation is in force for techniques exploiting microscopic moiré gratings[5,6] and time average geometric moiré methods.[7] The ability to exploit the natural microstructure of the surface as a stochastic grating would eliminate this restraint. This paper tries to show that such techniques can be developed and successfully applied for analysis of dynamic displacements.

ONE-DIMENSIONAL EXAMPLE

A one-dimensional system is analyzed for simplicity. Let the grayscale surface intensity I be defined as a function of x. It is assumed that values of function $I(x)$ vary in the range between 0 (black colour) and 1 (white colour). If the analyzed non-deformable body oscillates harmonically then the instantaneous intensity of illumination at location x_0 and time t will be (Fig. 1):

$$I(x_0 - a\ \sin(\omega t + \varphi)) \tag{1}$$

where a = amplitude; ω = frequency; φ = phase of oscillations. If a time average technique is used for the determination of surface intensity (and the exposure time is much longer than the frequency of oscillations) then the intensity of illumination at x_0 will be:

$$A(x_0) = \lim_{T\to\infty} \frac{1}{T} \int_0^T I(x_0 - a\ \sin(\omega t + \varphi))dt \tag{2}$$

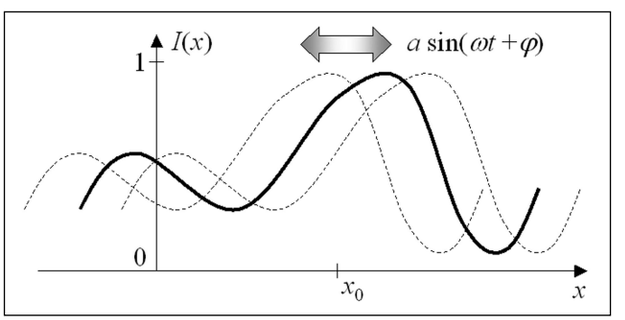

Fig. 1: One-dimensional example

where T = exposure time.

In case of a regular moiré grating $I(x)$ can be expressed as a harmonic function:[5]

$$I(x) = \cos^2\left(\frac{\pi}{\lambda}\, x\right) \tag{3}$$

where λ = the parameter defining the pitch of the grating. Then the time average intensity will be:[5]

$$A(x_0) = \lim_{T\to\infty} \frac{1}{T} \int_0^T \cos^2\left(\frac{\pi}{\lambda}\,(x_0 - a\ \sin(\omega t + \varphi))\right)dt$$

$$= \frac{1}{2} + \frac{1}{2}\cos\left(\frac{2\pi}{\lambda}\, x_0\right) \mathrm{J}_0\left(\frac{2\pi}{\lambda}\, a\right) \tag{4}$$

where J_0 is the zero order Bessel function of the first kind. It can be noted that the time average geometric moiré fringe pattern does not carry any information either on the frequency or on the phase of oscillations.

STOCHASTIC MOIRÉ GRATING

The fringe pattern produced by a regular geometric moiré grating plotted on the surface of a flat deformable body can bring valuable information about the field of dynamic displacements when time averaging techniques are used.[5] The condition for formation of fringes is that the amplitudes of oscillation must be different in different parts of the analyzed deformable body. The regular grating will produce no interference fringes if the body oscillates as a solid non-deformable body. What would happen if the moiré grating is not a regular (function $I(x,y)$ and is any function bounded in [0;1])? If the analyzed body would oscillate as a deformable body, the interpretation of the generated image would be extremely complicated. In general, the problem of reconstruction of the field of dynamic displacements would not have a unique solution.

On the other hand, if a regular moiré grating with microscopic pitch is formed on a surface of a deformable body, and only a small part of the object is analyzed using time average techniques, it is hard to expect any valuable results due to the fact that a small microscopic area will move like a solid non-deformable body. But if a non-regular moiré grating is formed on the surface of the solid oscillating non-deformable body, could the generated time average image be somehow interpreted? And if the images can be interpreted, how could

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IDENTIFICATION OF IN-PLANE VIBRATIONS

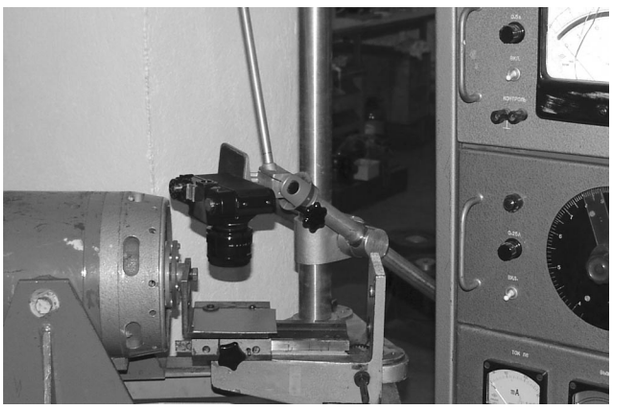

Fig. 2: General view of the experimental set-up

the non-regular moiré gratings be manufactured on the surface of micro-objects?

The answer to the last question is straightforward—the stochastic microstructure of the surface of the analyzed object can serve itself as a stochastic grating. Grayscale levels of the intensity of the surface can serve perfectly for that purpose. Firstly, no expensive microscopic grating formation is necessary. Secondly, a regular moiré grating can detect dynamic displacements only in the direction orthogonal to the grating lines. A stochastic grating has no definite line directions; therefore it has no previously mentioned limitations.

DESCRIPTION OF THE EXPERIMENTAL TECHNIQUE

In order to prove the applicability of the discussed technique a simple experimental set-up was developed comprising a high-resolution digital camera placed on top of a vibrating steel plate (Fig. 2). The steel plate was covered with a layer of paint for better contrast of the stochastic grating.

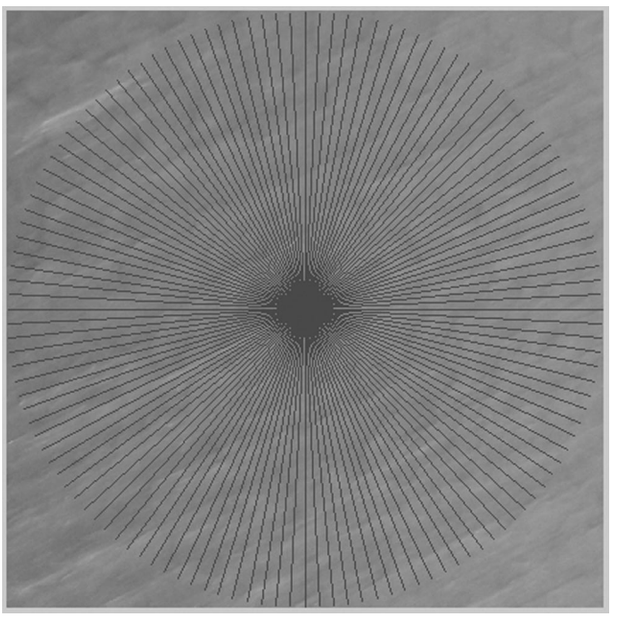

Fig. 4: Polar cutting diagram

Two experimental images are shown in Fig. 3—the surface of the plate in a state of equilibrium and a time average image of the vibrating plate (exposure time 3 s at frequency of excitation 200 Hz).

Step 1—Detection of the Direction of Vibration

The first task is to determine the direction of vibration from the experimental time average image. Therefore the experimental image is cut in different directions (Fig. 4). In each cut the variation of the intensity is reconstructed as a one-dimensional function (Fig. 5). Standard 256 grayscale discrete levels of intensity are used (0—black; 255—white).

The idea of the algorithm is to determine such a direction in which the intensity of illumination cannot vary a lot in a

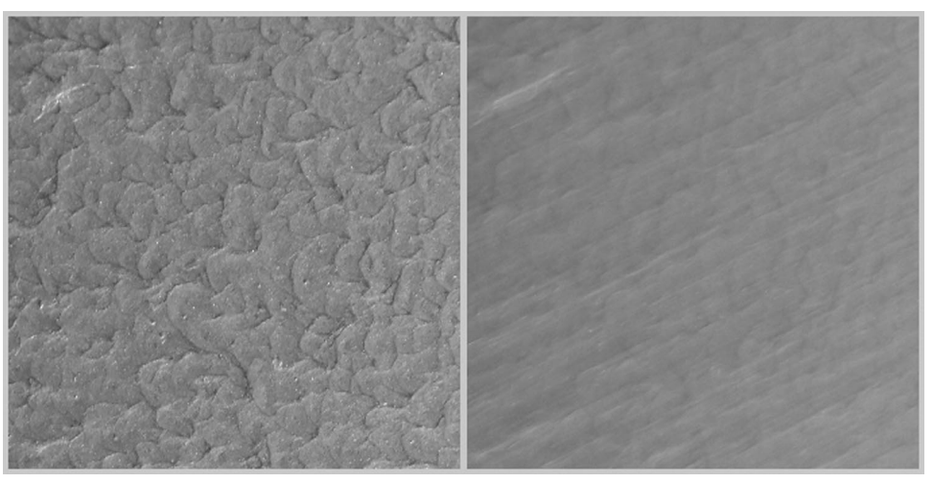

Fig. 3: Plate in the state of equilibrium (left) and its vibration mode (right)

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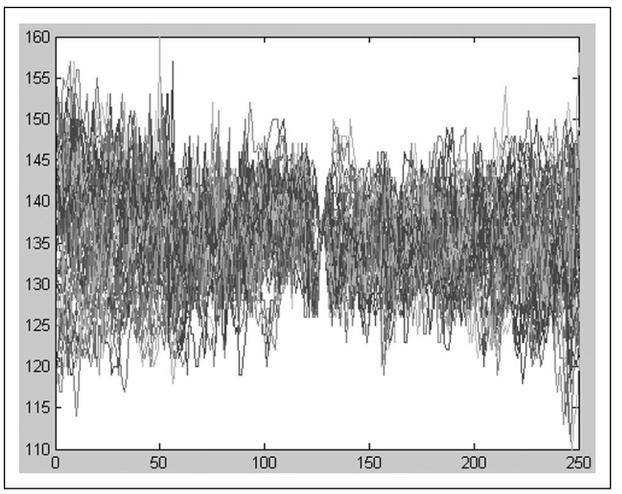

Fig. 5: Intensity variation in different cut directions (*x* axis represents the pixel number; *y* axis—intensity)

region of a small interval. There is no restriction for large variation in long intervals—the algorithm detects image blurring caused by uni-directional vibrations. One-dimensional intensity functions in different cut directions are split into 50 subintervals each, and the intensity variation in each interval is determined (Fig. 6). It can be noted that only 2 directions are represented in Fig. 6 for the clarity of presentation. The bottom histogram shows the intensity variation in each subinterval in both directions.

Next the average intensity variation in every direction is calculated. The results are presented in a polar diagram where the angle of cut varies from 0 to π and the radii denote average variations of intensity in appropriate directions (Fig. 7).

It is clear that the minimum average intensity variation corresponds to the 8-th pin counter clockwise in Fig. 7; this suggests that the direction of oscillation is $\pi/8$.

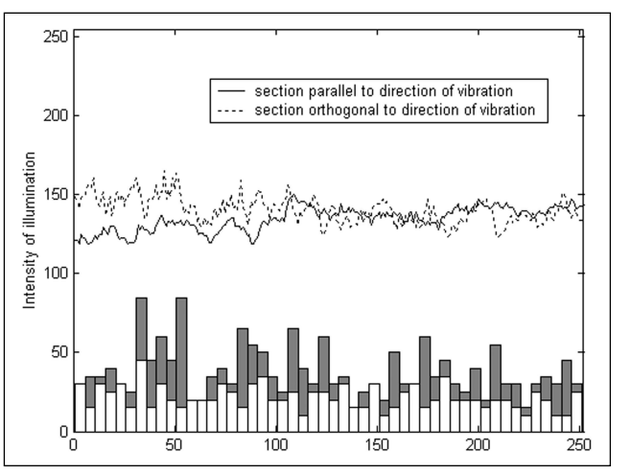

Fig. 6: Detection of the direction of oscillation

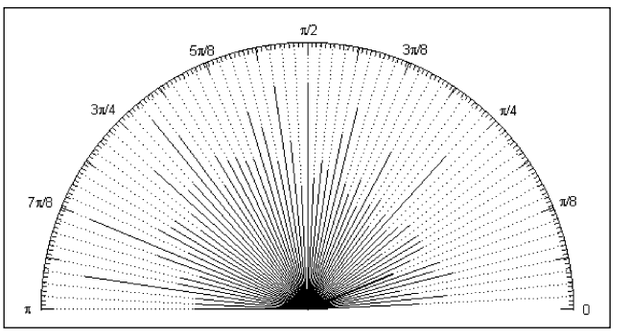

Fig. 7: Polar diagram of average intensity variation in different directions

Step 2—Determination of the Amplitude of Vibration

Determination of the amplitude of vibration when the direction of the vibration is known is an inverse engineering problem. Unfortunately, the solution of this problem is not unique when the function $I(x, y)$ is unknown. Further solution of the problem requires the image of the surface in the state of equilibrium.

Digital image processing techniques are used to simulate the time average vibrating image from the static image. MATLAB Image Processing Toolbox[8] procedures are used for image manipulation. 256 discrete locations in a period of harmonic motion are used to build the averaged digitally mimicked image at a pre-determined amplitude of oscillation. Then the difference between the mimicked image and the experimental time average image is calculated. The mean intensity of the produced image shows how close the reconstructed amplitude is to the physical amplitude. Ideally the difference image should be a black area. In practice it is hard to expect to get a zero average intensity of the difference image due to accumulation of errors. Those errors could be caused by the fact that the motion of the surface is not strictly one-directional, the static and time average experimental images can be misaligned, the reconstruction of the direction of the oscillation is not ideally accurate (limited number of discrete directions), etc.. Nevertheless, if the simulations are performed for different amplitudes, the minimum mean intensity of the difference image will serve as a good estimate of the experimental amplitude of the in-plane vibrations.

The process of the minimization of the mean intensity of the difference image is presented in Fig. 8. The reconstructed amplitude of oscillation is 0.78 mm.

Numerically simulated images and the difference images at different amplitudes are presented in Fig. 9.

PHYSICAL LIMITS OF THE TECHNIQUE

The described method for identification of vibration parameters from time average experimental pictures has certain limits in the sense of maximum detectable amplitude. If the size of the static image is $L_x \times L_y$ and the angle of the direction of vibration with the x axis is α, then the size of the

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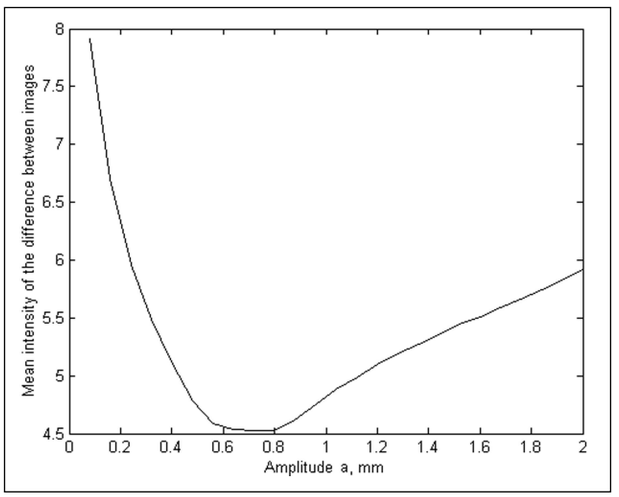

Fig. 8: Reconstruction of the amplitude of vibration

numerically reconstructed time average image which can be used for image subtraction will be $(L_x - 2a\cos\alpha) \times (L_y - 2a\sin\alpha)$ when $0 \leq \alpha < \pi/2$ and $(L_x + 2a\cos\alpha) \times (L_y - 2a\sin\alpha)$ when $\pi/2 \leq \alpha < \pi$. The smaller is the usable part of the reconstructed image, the higher is the uncertainty of the algorithm. This uncertainty η can be expressed as a ratio between the area of useless image and the total area of the image:

$$\eta(L_x,L_y,a,\alpha) = \begin{cases} 2a\left(\dfrac{\sin\alpha}{L_y} + \dfrac{\cos\alpha}{L_x}\right) - \dfrac{2a^2}{L_xL_y}\sin(2\alpha) \;\; when \;\; 0 \leq \alpha < \pi/2 \\ 2a\left(\dfrac{\sin\alpha}{L_y} - \dfrac{\cos\alpha}{L_x}\right) + \dfrac{2a^2}{L_xL_y}\sin(2\alpha) \;\; when \;\; \pi/2 \leq \alpha < \pi \end{cases} \tag{5}$$

where $0 \leq \alpha < \pi$; $a < \min(L_x;L_y)$. The Omni-directional average uncertainty can be calculated as:

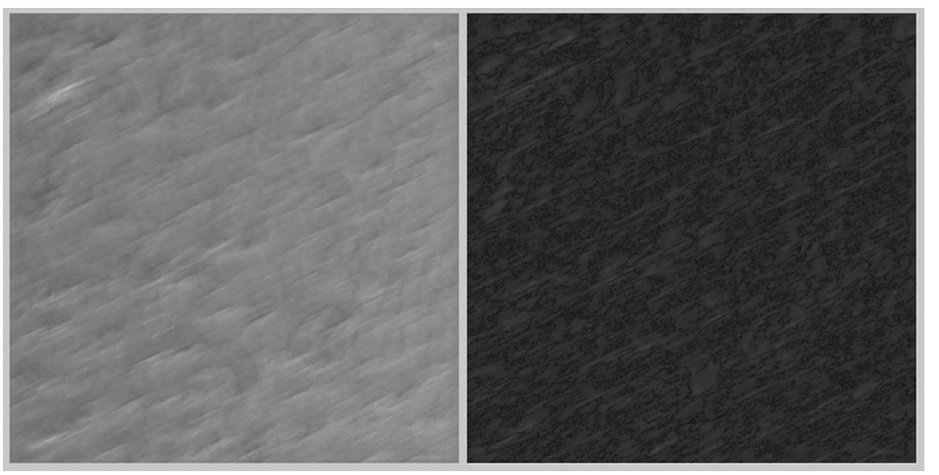

Fig. 9a: Simulated (left) and difference (right) images at $a = 0.4$ mm

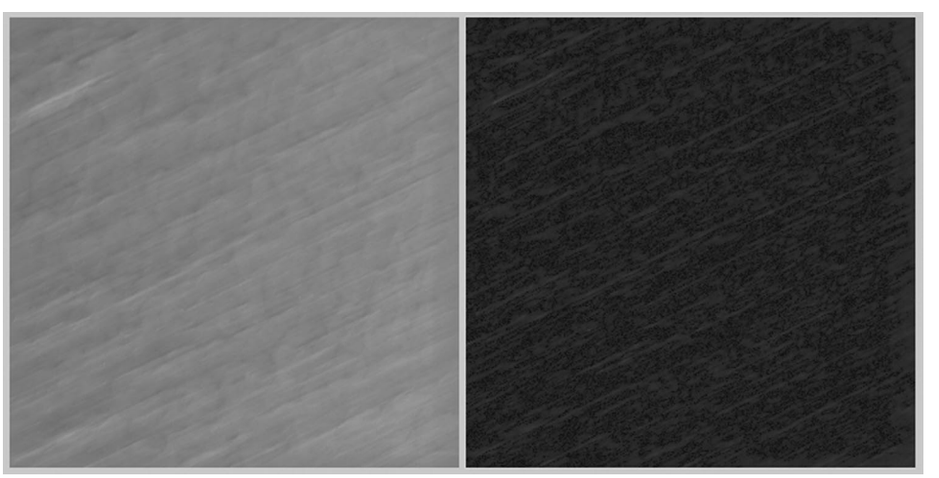

Fig. 9b: Simulated (left) and difference (right) images at $a = 0.78$ mm

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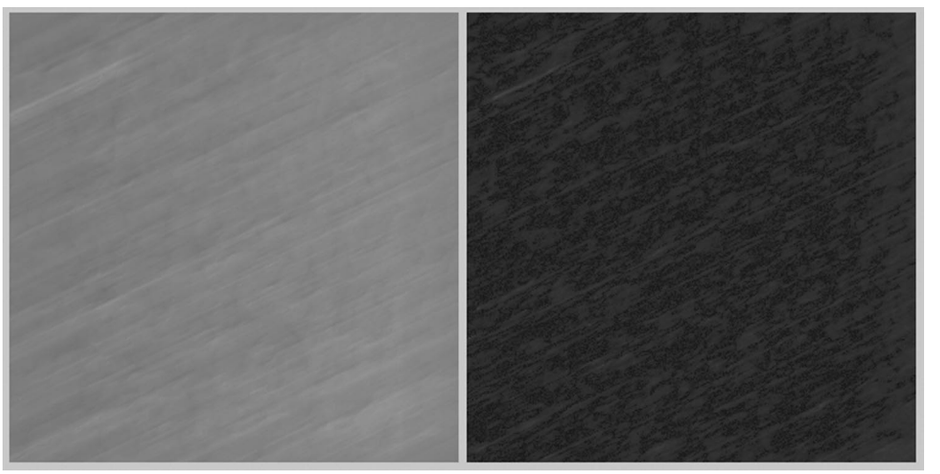

Fig. 9c: Simulated (left) and difference (right) images at $a = 1.2$ mm

$$\overline{\eta(L_x,L_y,a)} = \frac{1}{\pi}\int_0^{\pi} \eta(L_x,L_y,a,\alpha)d\alpha$$

$$= \frac{4a}{\pi}\left(\frac{1}{L_x}+\frac{1}{L_y}\right) - \frac{4a^2}{\pi L_x L_y} \qquad (6)$$

It is hard to expect that the reconstructed amplitude of vibration is accurate if the uncertainty is greater than 0.5. For simplicity it is assumed that $L_x = L_y = L$ and the solution of the inequality $\overline{\eta(L,L,a)} < 0.5$ leads to the following result:

$$0 \le a < (1 - \sqrt{1 - \pi/8})L \approx 0.22\,L \qquad (7)$$

CONCLUDING REMARKS

The proposed technique for identification of parameters of in-plane vibrations is based on digital image processing techniques. Identification of the direction of vibrations from experimental images enables the construction of the simulated time average digital images. That builds the ground for the development of the amplitude reconstruction technique, which is based on the minimization of the mean intensity of the differences between the experimental time average image and the numerically simulated images.

The presented technique is applied for reconstruction of parameters of in-plane vibrations when these vibrations are uni-directional. But this fact does not lessen the practical value of the method. The motion of any sufficiently small region of a linear elastic body performing single mode resonance oscillations can be interpreted as uni-directional vibrations. Elliptic motions may occur in case of propagating waves, multi-mode excitation or non-linearity of the system. In such cases the technique can be appropriately adapted.

This technique is a natural extension of time average geometric moiré with regular grating and can be effectively exploited in the analysis of vibrating microstructures.

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