TECHNIQUES

by M. Ragulskis, L. Ragulskis, and R. Maskelianas

APPLICABILITY OF TIME AVERAGE GEOMETRIC MOIRÉ FOR VIBRATING ELASTIC STRUCTURES

Double exposure geometric moire[1] can produce valuable and meaningful patterns of fringes. It is a well-established technique with various applications.[2] There exist numerous technical solutions for dynamic moiré analysis.[2] A typical example is when a free surface of liquid is illuminated through a grid focusing an array of light and dark lines on the surface. Interpretation of the dynamic picture (or series of static pictures) of grating can bring useful information about the motion of the surface. Another widely used application is the analysis of transient mechanics of deformation exploiting laser moiré interferometry.[3,4,5]

Nevertheless, direct dynamic interpretation of structural vibrations from dynamic pictures of a moiré grating is hardly possible due to a relatively high frequency of resonant oscillations of the analyzed structures. An alternative could be the coupling of time average techniques and geometric moiré fringe analysis. It can be noted that direct geometric moiré analysis (not laser moiré interferometry) is considered. Yet there are few open questions. Will the patterns of fringes be formed as the time average techniques are used? And, if the pattern of fringes is formed, how can it be interpreted? This paper tries to answer these questions and prove the applicability of time average geometric moiré for analysis of vibrating elastic structures. Such methods of measurement do not require a complex experimental set-up and can provide valuable results in many practical applications.

DOUBLE EXPOSURE GEOMETRIC MOIRÉ

For simplicity, a one-dimensional system is analyzed first.

The moiré grating in the state of equilibrium is defined as a harmonic variation of grey-scale in the range between 0 (representing black color) and 1 (representing white color):

$$I_1(x) = \frac{1}{2}\left(1 + \cos\left(\frac{2\pi}{\lambda}x\right)\right) = \cos^2\left(\frac{\pi}{\lambda}x\right), \qquad (1)$$

where I_1 stands for the grey scale level of the surface in a state of equilibrium; x—the longitudinal co-ordinate; and λ—the parameter defining the pitch of the grating. It can be noted that the definition of such a grating is well suited for numerical reconstruction of moire fringes.[6] The intensity is a continuous function, where line width is 50 % of the pitch.

The grating of fringes in the deformed state is defined as:[6]

$$I_2(x, u) = \frac{1}{2}\left(1 + \cos\left(\frac{2\pi}{\lambda}(x - u)\right)\right) = \cos^2\left(\frac{\pi}{\lambda}(x - u)\right), \qquad (2)$$

where I_2 is the intensity of the grey scale level for the deformed state; and u—the displacement from the state of equilibrium at point x. Such formulation of intensity enables straightforward embedding of the results from finite element analysis to digital computer plotting procedures.

Superposition of two gratings (double exposure of the gratings in the state of equilibrium and in the deformed state) produces a moiré fringe pattern:

$$I_s(x, u) = \frac{I_1(x) + I_2(x)}{2} = \frac{1}{2} + \frac{1}{2}\cos\left(\frac{2\pi}{\lambda}\left(1 - \frac{u}{2x}\right)x\right) \cdot \cos\left(\frac{\pi}{\lambda}u\right), \qquad (3)$$

where I_s stands for the intensity in double exposure geometric moiré. The centers of the fringes are located at $u = \lambda\left(n + \frac{1}{2}\right)$, and the intensity is 0.5 at the center of the fringes. Each new fringe indicates an increase or decrease of specimen displacement by one grating pitch (component of the displacement perpendicular to the reference grating lines). If the displacement from the state of equilibrium u is assumed to be a linear function from x:

$$u = kx, \qquad (4)$$

where k is the coefficient of proportionality, then

$$I_s(x) = \frac{1}{2} + \frac{1}{2}\cos\left(\frac{2\pi}{\lambda}\left(1 - \frac{k}{2}\right)x\right) \cdot \cos\left(\frac{\pi}{\lambda}kx\right), \qquad (5)$$

and the resulting intensity distribution takes the form of beatings. Such effect can be successfully exploited in procedures building computer generated moiré fringes from finite element calculations.[6]

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Let's consider the previously mentioned one-dimensional model with the assumption that an elastic structure performs harmonic oscillations and the amplitudes (maximum displacements from the status of equilibrium) are $u(x)$. If the time of exposure is much longer than the period of oscillations, the resulting pattern will be formed as an integral sum of the continuous process of motion:

$$I_a(x, u) = \lim_{T\to\infty}\frac{1}{2T}\int_0^T \left(1 + \cos\left(\frac{2\pi}{\lambda}(x - u\sin(\omega t + \varphi))\right)\right)dt = \frac{1}{2} + \frac{1}{2}\cos\left(\frac{2\pi}{\lambda}x\right)J_0\left(\frac{1\pi}{\lambda}u\right) \qquad (6)$$

where I_d stands for the intensity in time average geometric

M. Ragulskis is affiliated with Kaunas University of Technology, Kaunas, Lithuania. L. Ragulskis is affiliated with Vytautas Magnus University, Kaunas, Lithuania. R. Maskeliumas is affiliated with Vilnius Gediminas Technical University, Vilnius, Lithuania.

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moiré analysis; J_0—zero order Bessel function of the first kind; T—exposure time; ω—angular frequency of oscillations; and φ—phase. It can be noted that integral $\lim_{T\to\infty} \frac{1}{2T} \int_0^T \sin\left(\frac{2\pi}{\lambda}(u\,\sin(\omega t + \varphi))\right) dt$ in Eq. 6 converges to zero due to the unevenness of the sine function.

Of course, J_0 is not a periodic function, but the roots of Eq. $J_0\left(\frac{2\pi}{\lambda}u\right) = 0$ are in some sense comparable with the roots of the expression $\cos\left(\frac{2\pi}{\lambda}u\right) = 0$. That leads to an important conclusion. That is, the number of fringes for the same field of deformations is almost double in a time average geometric moiré picture if compared to a double exposure geometric moiré picture (due to the term $\cos\left(\frac{\pi}{\lambda}u\right) = 0$ in Eq. (3)).

NUMERICAL EXPERIMENTS

A numerical technique for reconstruction of a pattern of moiré fringes is based on the following approximation:

$$I_a(x,u) = \frac{\omega}{2\pi} \int_{-\frac{\varphi}{\omega}}^{\frac{2\pi-\varphi}{\omega}} \cos^2\left(\frac{\pi}{\lambda}(x - u\,\sin(\omega t + \varphi))\right) dt \approx \frac{1}{m}\sum_{i=1}^{m}\left\{\cos^2\left(\frac{\pi}{\lambda}\left(x - u\,\sin\left(\frac{2\pi}{m}(i-1)\right)\right)\right)\right\} \tag{7}$$

The first equality in Eq. (7) follows from the periodicity of the harmonic function. The second approximate equality builds the foundation for an approximate numerical technique of the reconstruction of the pattern of fringes. The quality of the reconstruction gets better the higher the value of m (the number of discrete states in a period of oscillations). It can be noted that neither ω nor φ has any effect to the pattern of fringes.

A two-dimensional elastic body in a state of plane stress is analyzed. A finite element mesh in its equilibrium state and in the state of maximum deformation in its second eigenmode is illustrated in Fig. 1.

Superposition of moiré gratings at the state of equilibrium and the state of maximum deformation produces double exposure fringe patterns at horizontal and vertical gratings, which are illustrated in Fig. 2 and Fig. 3. The effect is shown while using the technique for building digital images in a virtual projection plane.[7]

Time average moiré fringes are plotted for the same plate. The number of discrete positions in a period of harmonic oscillations is 64. Produced patterns are presented in Fig. 4 and Fig. 5. It can be noted, that the proposed technique of visualisation works well on rather coarse finite element meshes and is scalable for parallel computations.

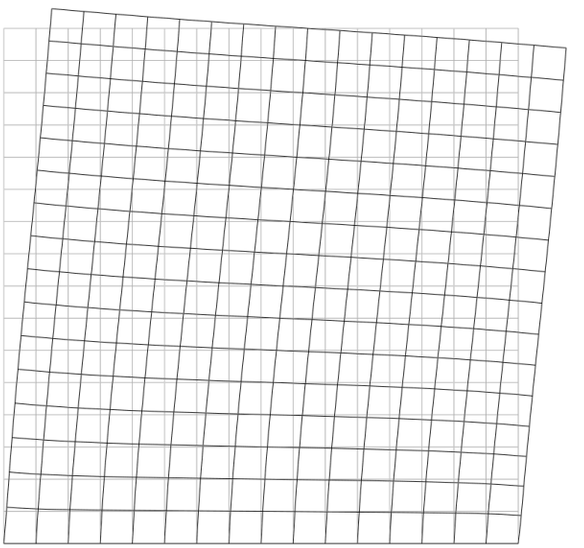

Fig. 1: Finite element mesh showing the equilibrium state (grey lines) and the state of maximum deformations (black lines)

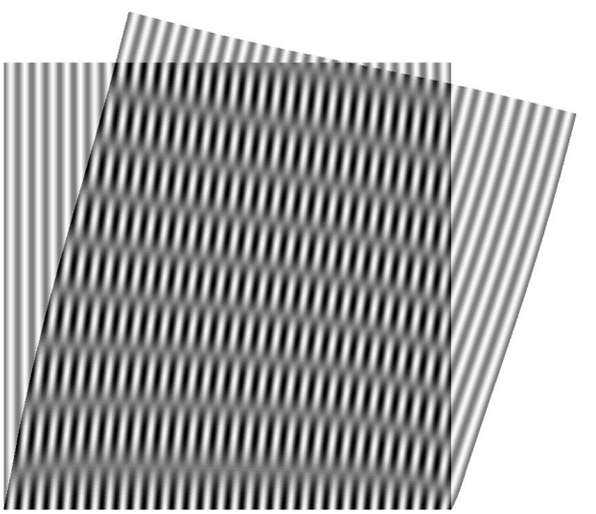

Fig. 2: Double exposure moiré fringes (vertical grating)

Fringe patterns in time average moiré pictures prove that the number of fringes in time average moiré images is nearly double if compared to double exposure moiré images, though the parameters of the analyzed structure are identical. Good correlation is shown between analytical and numerical investigations, which validates them perfectly. A drawback in the sense of interpretation of results, is the fact that the higher fringes are blurred due to the Bessel function term in Eq. 6.

EXPERIMENTAL INVESTIGATIONS

The goal of experimental time average moiré analysis is to validate the whole idea of the method. A grating of dark and white lines is drawn on the surface of rubber specimens, which are mounted to a shaker system (Fig. 6). One rubber specimen is produced with a horizontal, and another with a

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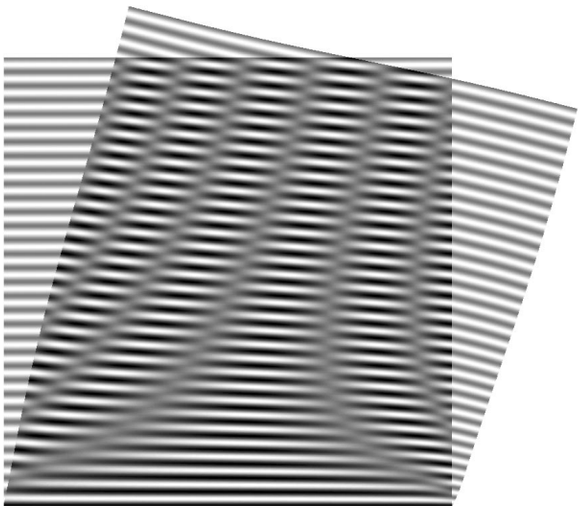

Fig. 3: Double exposure moiré fringes (horizontal grating)

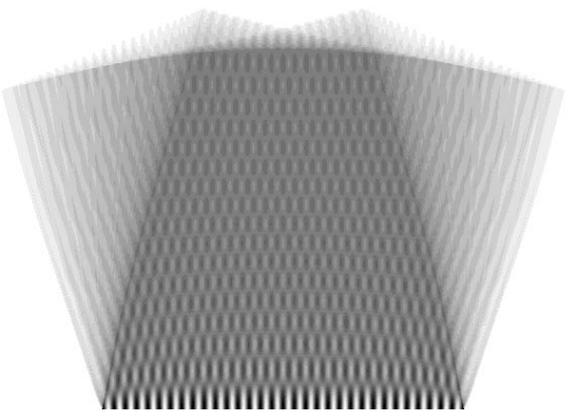

Fig. 4: Time average moiré fringes (vertical grating)

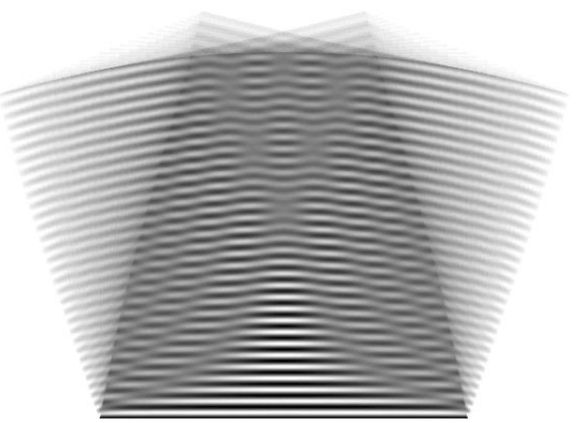

Fig. 5: Time average moiré fringes (horizontal grating)

vertical grating. The grating pitch is 1 mm, the line width is half of the pitch, and the driving frequency of the shaker table is 51 Hz, which corresponds to the second resonance frequency of the rubber specimens. A digital camera with exposure time $T = 1.5$ seconds is used. Produced images are

Fig. 6: View of the experimental set-up

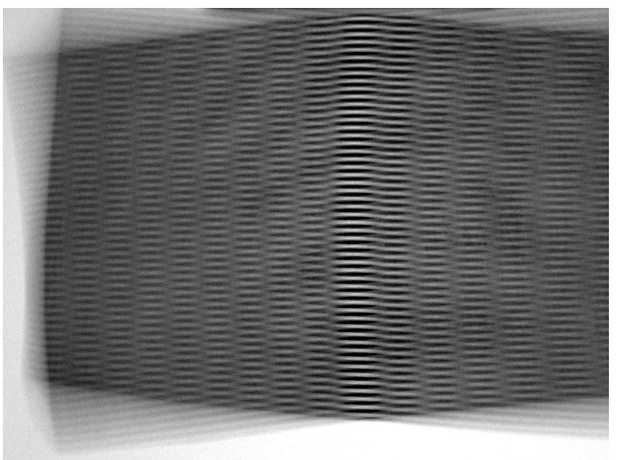

Fig. 7: Experimental time average moiré fringes (vertical grating)

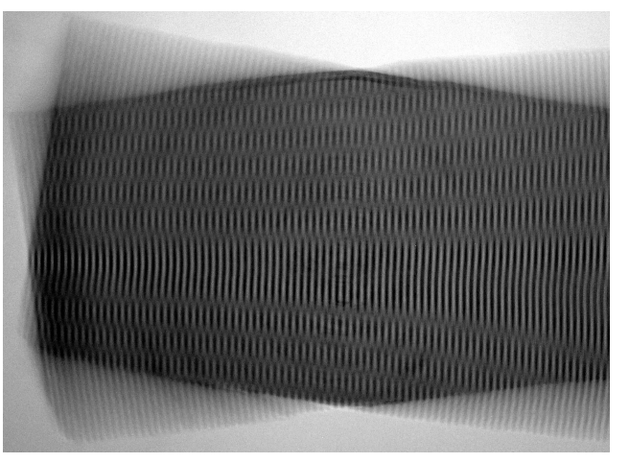

Fig. 8: Experimental time average moiré fringes (horizontal grating)

presented in Fig. 7 and Fig. 8. It can be noted that a high sharpness of grating is preserved in the motionless zones of the rubber specimen. The fringes can be identified with a naked eye during a live experiment, though they are blurred at higher fringe numbers.

CONCLUDING REMARKS

Time average geometric moiré for analysis of in-plane vibrations of elastic structures is proposed in this paper. The intensity of the higher fringes is blurred, which hides this method among other non-contact whole-field experimental techniques for analysis of structural vibrations.

This paper proves that such a method is feasible for the analysis of vibrations. It has many advantageous features and is a straightforward method that does not require a complex experimental set-up. It is even simpler than double exposure geometric moiré since no artificial superposition of the reference image and the image of the object in the deformed state is necessary. Naturally, the interpretation of fringe patterns in time average geometric moiré is different from the double exposure geometric moiré. But this fact does not lessen the practical value of the method, which can be effectively exploited for analysis of vibrating elastic structures.

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