Order Adaptive Quadrature Rule for Real Time Holography Applications

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Abstract. Order adaptive algorithm for real time holography applications is presented in this paper. The algorithm is based on Master-Worker parallel computation paradigm. Definite integrals required for visualization of fringes are computed using a novel order adaptive quadrature rule with an external detector defining the order of integration in real time mode. The proposed integration technique can be effectively applied in hybrid numerical-experimental techniques for analysis of micro-mechanical components.

Keywords: Order adaptability, real time integration, holography.

1 Introduction

Holographic interferometry [1] is a powerful experimental technique for analysis of structural vibrations, especially if the amplitudes of those vibrations are in the range of micrometers. Recent advancements in optical measurement technology [2] and development of hybrid numerical-experimental techniques [3] require application of computational algorithms not only for post-processing applications like interpretation of experimental patterns of fringes, but embedding real time algorithms into the measurement process itself [4].

Computation and plotting of patterns of time average holographic fringes in virtual numerical environments involves such tasks as modelling of the optical measurement setup, geometrical and physical characteristics of the investigated structure and the dynamic response of the analysed system [5]. Calculation of intensity of illumination at any point on the hologram plane requires computation of definite integrals over the exposure time. If the analysed structures perform harmonic oscillations that do not impose any complications – there exist even analytical relationships between the intensity of illumination, amplitude of oscillation, laser wavelength, etc. [1]. But if the oscillations of the investigated structures are non-harmonic (what is common when structures are nonlinear) and the formation of patterns of fringes is implemented in real time mode, the calculation of definite integrals becomes rather problematic. The object of this paper is to propose an order adaptive algorithm which could be effectively applicable for calculation of definite integrals in different real time applications.

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2 Integration Rule Without Limitation for the Number of Nodes

Higher order Newton-Cotes quadrature formulas [6] require that the number of nodes must be a divisible numeral. For example the second order Newton-Cotes rule already requires that the number of nodes must be odd. Such conditions mean that a significant number of nodes at the end of an experimental time series must be deleted and the integration interval artificially shortened for higher order Newton-Cotes rule, if the number of nodes is not known at the beginning of the experiment. Therefore there exists a definite need for a high order integration rule with a constant time step without any requirement for the number of time steps. Such quadrature formula is proposed in [7]:

$$\int_{t_0}^{t_0+(k-1)h} f(t)\,dt = \left(\sum_{i=1}^{m} a_i f_i + \sum_{i=1}^{k-2m} f_{m+i} + \sum_{i=1}^{m} a_{m-i+1} f_{k-m+i} \right) h, \qquad (1)$$

where a_i are the weights and f_i are the discrete values of sampled function f at time moments $t_0 + (i-1) \cdot h$, $i = 1, \ldots, k$. It has been proved that this integration rule is exact when the integrated function is a polynomial of the m-th order, if only m is odd [7]. The numerical values of the weights a_i are presented in the Table 1 at different values of m. The parameter p in this table denotes the maximum order of exactly integrated polynomials; l is the order of the error term expressed in the form $O(h^l)$.

Table 1. Nodal weights of the integration rule

m	2	3	4	5	6	7
a_1	0.5	0.37500000	0.33333333	0.32986111	0.31875000	0.30422454
a_2	1	1.1666667	1.2916667	1.3208333	1.3763889	1.4603836
a_3		0.95833333	0.83333333	0.76666667	0.65555556	0.45346396
a_4			1.0416667	1.1013889	1.2125000	1.4714286
a_5				0.98125000	0.92569444	0.73939319
a_6					1.0111111	1.0824735
a_7						0.98863261
p	1	3	3	5	5	7
l	2	4	4	6	6	8

It can be noted that finite element method was used for the derivation of the proposed quadrature rule which can be interpreted as a new variant of Gregory type formulas [6]. Unfortunately, the proposed quadrature rule (also Gregory type rules) can be used only when the order is predefined before the experiment and does not change over the integration process. This paper proposes a multi-processor parallel algorithm with full order adaptability in real time calculation mode.

3 The Basic Real Time Integration Rule

Let's suppose that function f is sampled starting from t_0 at equally spaced time steps; the length of a time step is h. Due to the real time process the number of nodes is not predefined before the experiment and process continues until the end of the sampling. Let's suppose that the terminal moment of the sampling occurs at $t_0 + 7h$ (8 function values f_i, $i = 1, \ldots, 8$ are produced during the sampling process). Order of the integration rule is predetermined to be $m = 3$.

1. The first sum on the right side of eq. (1) is computed:

$$Sum_1 = a_1 f_1 + a_2 f_2 + a_3 f_3, \tag{2}$$

where $a_1 = 0.375, a_2 = 1.1666667, a_3 = 0.95833333$ (Table 1).

2. Starting from the fourth node, the following sum is computed until the end of the time series:

$$Sum_2 = f_4 + f_5 + f_6 + f_7 + f_8. \tag{3}$$

3. When the sampling is terminated, reverse computation of the third sum of eq. (1) is done:

$$Sum_3 = (a_3 - 1)\, f_6 + (a_2 - 1)\, f_7 + (a_1 - 1)\, f_8. \tag{4}$$

4. Finally, the definite integral $\int_{t_0}^{t_0+7h} f(t)\, dt$ is calculated according to eq. (1):

$$I = (Sum_1 + Sum_2 + Sum_3)\, h. \tag{5}$$

The process can terminate at any time step, if only $k \geq 2m$, but the last three values of the sampled function must be saved at every time moment in order to calculate Sum_3.

Now we will generalize the presented example for $m-$th order integration rule, if only the minimum number of nodes *is* $2m$. The algorithm is based on Master-Worker paradigm [8]. Schematic graphical representation in Fig. 1 helps to interpret the computation process.

Several notations used in Fig. 1 can be explained in more detail. Order of the integration rule m is predefined before the experiment. Calculation of Sum_1 is performed by Master processor (grey right arrow in signal diagram; block n in time diagram and node n in flow chart diagram). After m terms are included into Sum_1, the Master processor continues summation of nodal values of the integrand until the sampling process is terminated (white right arrow in signal diagram; block $n^{(1)}$ in time diagram and node $n^{(1)}$ in flow chart diagram). When the sampling is over, Worker processor performs reverse calculation of Sum_3 (grey left arrow in signal diagram; block $\bar{n}$ in time diagram and node $\bar{n}$ in flow chart diagram).

It can be noted that the last m values of the sampled function must be remembered at every time node in order to calculate Sum_3.

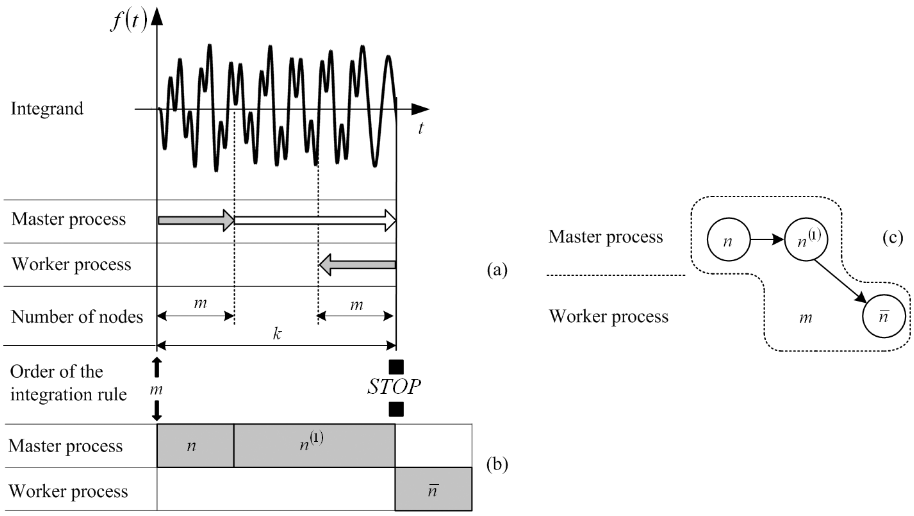

Fig. 1. Schematic representation of the basic model: (a) signal diagram; (b) time diagram; (c) flow chart diagram

4 Order Adaptive Algorithm for Real Time Applications

The presented basic real time integration rule copes well with integrands which can be approximated by a polynomial of a definite order in the domain of integration. But if the variation of the integrand is fast in some regions and slow in another regions, then order adaptability should be used to increase the accuracy of a definite integral. One can suggest to select very large m at the beginning of the experiment, but then we may face the risk that $k < 2m$.

We assume that there exists a detector which measures the values of the integrand and recommends the order of integration rule at any time moment in the domain of integration. Let's assume that the present order is m_1 and the detector recommends order m_2. Then two different situations may occur. If the number of sampled nodes since m_1 was declared is higher than or equal to $2m_1$, the transition to order m_2 can be performed fluently. The Master processor starts calculating Sum_1 for order m_2, while Worker processor takes care for reverse calculation of Sum_3 for terminated m_1.

But if the number of sampled nodes since m_1 was declared is less than $2m_1$, the Worker processor cannot start reverse calculation of Sum_3 without damaging Sum_1. Therefore a much more complex transition to order m_2 takes place in this situation. If m_2 is higher than m_1 the Worker processor must return to the point where order m_1 was declared and must recalculate Sum_1 with order m_2. But the simplicity is misleading – the Master processor has already summated Sum_1 with order m_1 to the total sum! Therefore the Worker processor must evaluate different weighting coefficients for orders m_1 and m_2. Moreover, the length of the queue where the last function values are stored must be already not m_i, but $2m_i$ (here m_i is the current order).

If m_2 is lower than m_1, but the number of sampled nodes since m_1 was declared is less than $2m_1$, the integration with order m_1 must be continued until the number of nodes is equal to $2m_1$, and only after that the order m_2 can be accepted.

Finally, we may comment what would happen if the sampling process is terminated and the number of sampled nodes since m_i was declared is lower than $2m_i$. Unfortunately, there will be no any possible techniques to preserve order m_i (time step is constant and reverse sampling with smaller time step is impossible in real time mode). The only solution is to select maximum possible order for the available number of time steps (floored half of the number of time steps).

We will illustrate the described situations with the following example (Fig. 2 and Fig. 3).

One Master processor and two Worker processors are necessary for full real time mode. Algorithm control, integrand sampling and summation of sums Sum_1 and Sum_2 is performed by Master processor. The Worker processors run only when the order is changed. Worker processors send back the results to the Master processor.

As an extreme situation we describe the transition from order m_3 to order m_4 (Fig. 2) where the second Worker processor is necessary for real time integration. Master processor starts calculating Sum_1 (with weights corresponding to order m_3) as soon as the order m_3 is declared. The number of discrete time nodes

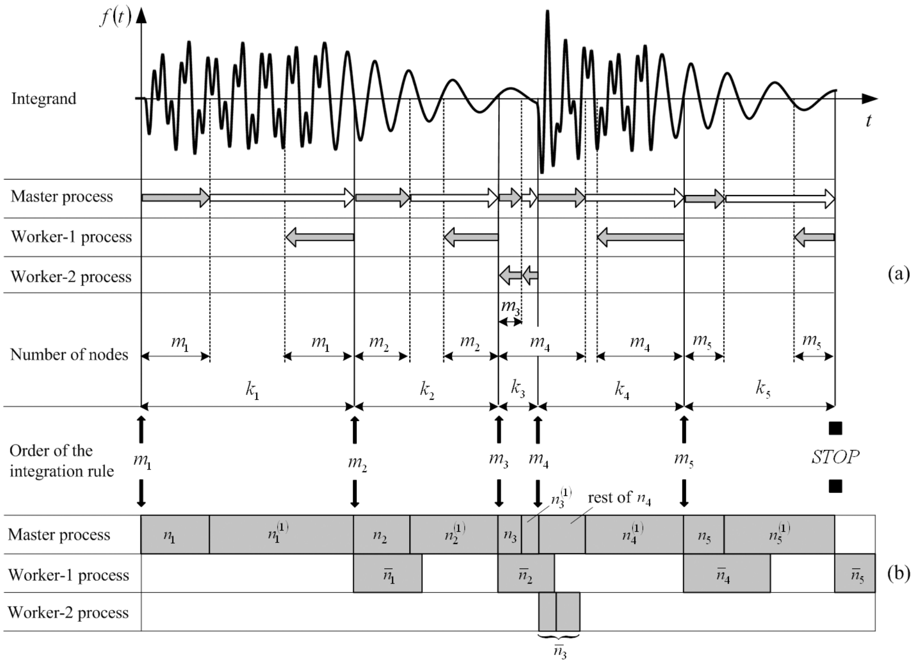

Fig. 2. Real time integration, general case: (a) signal diagram; (b) time diagram

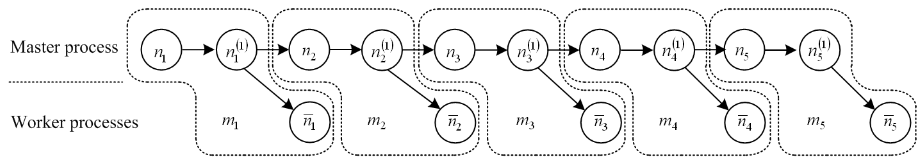

Fig. 3. Real time integration, general case: flow chart diagram

necessary for this procedure is m_3. As soon as Sum_1 is finished, Master processor starts summing non-weighted discrete function values. This process continues until order m_4 is declared. But the order detector has sensed a burst in the digital time series, so m_4 is much higher than m_3. In this particular situation we have that m_4 is even greater than k_3 (Fig. 2). Thus, the Worker processor must recalculate both the old Sum_1 and the rest non-weighted part (n_3 and $n_3^{(1)}$ in time diagram). Moreover, at the same time Sum_3 for order m_2 must be accomplished ($\bar{n}_2$ in time diagram). Thus Worker-2 processor is unavoidable for real time computation ($\bar{n}_2$ and $\bar{n}_3$ overlap in time diagram). The mathematical formulas for processes $\bar{n}_2$ and $\bar{n}_3$ (consisting from two parts) can be described explicitly in the text of the general algorithm which we omit due to the restrictions for the size of the manuscript.

5 Application of Real Time Integration Technique for Visualization of Holographic Interferograms

Computational visualization of holographic interferograms in virtual numerical environments is an important component of hybrid numerical – experimental techniques. These techniques are of crucial importance when the analysed systems perform non-harmonic motions what is a typical situation when micromechanical systems are considered [9].

Whenever a pattern of time average holographic fringes is considered, the intensity of illumination at the hologram plane is described by the following relationship [1]:

$$I\left(x,y\right)=\lim_{T\to\infty}\frac{1}{T^2}\left|\int_0^T \exp\left(j\frac{2\pi}{\lambda}\zeta\left(x,y,t\right)\right)dt\right|^2, \qquad (6)$$

where I is the intensity of illumination; T – exposure time; j – imaginary unit; λ – laser wave length; ζ – dynamic displacement at point (x,y) at time moment t. Usually the function $\zeta\left(x,y,t\right)$ is decomposed to a product of time function and coordinate function describing the modal shape. It is clear that accurate computation of definite integral in eq. (6) for finite exposure times is associated with the accuracy of pattern of fringes in the numerically reconstructed hologram.

Dynamic displacements of cantilevered micromechanical bar are presented in Fig. 4(a). Scanning laser is measuring the displacements at the marked nodes

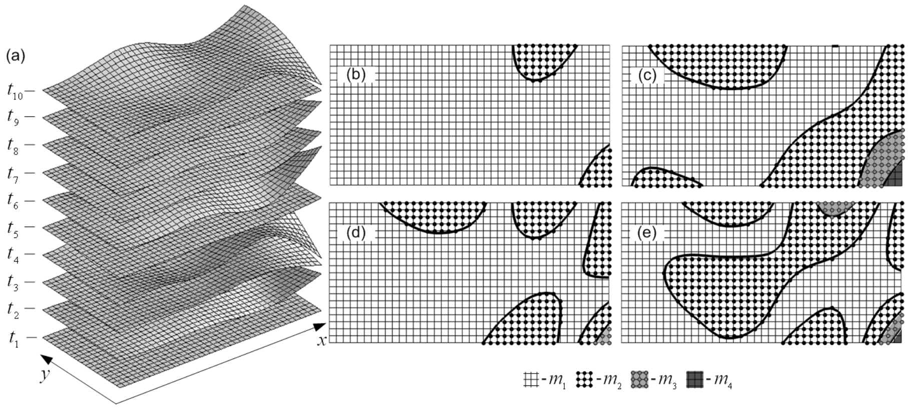

Fig. 4. Dynamic displacements of cantilevered micromechanical bar: (a) finite element shapes at different time moments; (b), (c), (d) and (e) – nodal orders of integration at different time moments

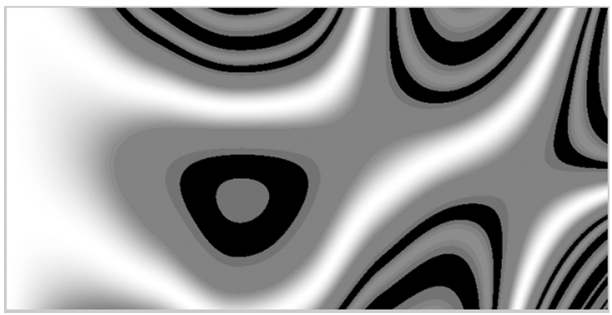

Fig. 5. Numerically reconstructed pattern of fringes

at discrete time moments. Intensity of illumination in the hologram plane is calculated at every node, so definite integrals are calculated at every node. The system is checking the magnitude of dynamic displacement at every node and generates the recommended order of integration which is based on the absolute value of discrete displacement at appropriate node. Figures 4(b), 4(c), 4(d) and 4(e) present the recommended orders of integration at different time moments; where $m_1 = 3$, $m_2 = 5$, $m_3 = 7$, $m_4 = 9$.

Figure 5 presents the produced time average holographic pattern of fringes.

6 Concluding Remarks

The presented procedure for real time calculation of definite integrals can be effectively applied in hybrid numerical-experimental techniques where time average intensities of illumination are reconstructed in virtual computational environment. Implementation of the proposed integration rule enables full real time computations with minimal data queue lengths and effective management of integration order.

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